THE OPEN UNIVERSITY OF TANZANIA

FACULTY OF SECIENCE, TCHNOLOGY AND ENVIRONMENTAL STDIES

FIRST ASSIGNMENT 2008 – 2009

OMT 305: TOPOLOGY

This assignment should be submitted by 12th December 2008

ANSWER ALL FOUR QUESTIONS

1. Let (Z, d_2), (X, d_1) and (X, d) be metric spaces. Prove that if (Z, d_2) is a subspace of (Y, d_1) and (Y, d_1) is a subspace of (X, d), then (Z, d_2) is a subspace of (X, d)..

2. (a) Define continuity of a function in terms of neighborhoods. Use the definition to prove that the composition of two continuous functions is a continuous.

   (b) Define the term "metric space".

   (c) Define continuity of a function in terms of metric spaces.

3. (a) Let X be a subset of $\Re^n$, the set of real numbers, and let F be a subset of X which is closed in X. Let also $x_1, \quad x_2, \quad x_3, \quad \ldots$ be a sequence of points of F which converges to a point $p \in X$. Prove that $p \in F$.

   (b) Let X be a non empty set. Let the mapping $d : X \times X \to \Re$ be a distance function on X defined, such that $d(x, y) = 1$ if $x \neq y$ and $d(x, y) = 0$ if $x = y$. Verify that the distance function d on X defines a metric on X.

4. Let $f : X \to Y$ be a function from a topological space X into a topological space Y.

   Let also A be the set of points of X where f is discontinuous. Let g be the restriction of f to $X \setminus A$. Is g continuous everywhere? Prove or disprove.

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SECOND ASSIGNMENT 2008/09

OMT 305: Topology

Hand-in by 27th March, 2009

Answer ALL Questions

1. Let X be any set and $\tau = 2^x$, Prove that (X, τ) is a topological space that contains the largest number of elements to any other topology that one may place on X .

2. (a) Let (X,T) be a topological space. Define a partition of (X,T).
   (b) When is a topological space (X,T) said to be connected?
   (c) Let X be a given set, $T_0 = \{\phi, T\}$ and D=$\{ \text{ A} \mid \text{A} \subseteq X \}$. Show that
   (i) (X, T_0) is connected. (ii) (X, D) can not be connected.

3. Let f be a mapping from a metric space X to another metric space Y .
   (a) If f is continuous then $f(\overline{A}) \subseteq \overline{f(A)}$ for all $A \subseteq X$.
   (b) If $D \subseteq X$ is dense, f is continuous and $f(X) = Y$ then $F(D)$ Iis dense in Y

4 Let X, Y and Z be topological space, and let $f : X \to Y$ and $g : Y \to Z$ be continuous functions. Prove that the composition $gof : X \to Z$ of the functions f and g is continuous.

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